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1. Introduction

Constant-market-shares (hereafter CMS) analysis is an accounting method for decomposing a country's aggregated export (share) growth into "total growth" (or "structural change") effects and "competitiveness" effects. The method, known as shift-and-share analysis in the empirical studies of structural changes in industrial and regional economics[1], became popular in applied international economics with the pioneering work of Tyszynski (1951) and it has been increasingly used and refined despite continued criticism both on theoretical and empirical aspects[2].

The main reasons for the popularity of CMS analysis are its simplicity and applicability to ready-to-use published data at a reasonable level of disaggregation. The basic analytic framework is an identity which defines the aggregated export level of a country as the sum of individual commodities exported to the rest of the world or to single foreign markets. This identity is decomposed into elements such as commodity shares and market shares of world trade, while its variations over time are disentangled into variations of aggregates of these components.

The method has therefore the nature of *ex post* analyses, like the structural analyses carried out on national macroeconomic accounts or input-output tables, and by no means can it provide by itself indications on the underlying determinants of the observed changes. Its usefulness is effectively synthesized by Magee (1975, p.221): "The technique reveals that, even if a country maintains its share of every product in every market, it can still have a decrease in its aggregate market share if it exports to markets that grow more slowly than the world average and/or if it exports products for which demand is growing more slowly than average".

Much of the early criticism was directed towards misunderstanding of the accounting role of this method, which was sometimes employed for "explaining" or forecasting purposes. However, the most important remarks were made by Baldwin (1958) and Richardson (1971a, 1971b), who considered CMS analysis as an index number approach in which different weights of aggregation can be chosen in order to obtain consistency in accounting for changes in total exports (or export shares). Richardson (1971a, 1971b) also noted that the values and sign of the various components may change if the final year of the period examined is considered as base year instead of the initial.

Moreover, in the discrete–time formulation of CMS a further element, apparently given by the interaction between commodity–composition and market–distribution effects, arises if the same base year is used for the weights of the various components. This "interaction" element was noted by Baldwin (1958) (following R. Lichtenberg's suggestions to a previous draft of his paper) and by Spiegelglas (1959) and was further discussed by Richardson (1971a, 1971b). There was not, however, a consensus on the interpretation of this residual element: some authors considered it a genuine economic variable (or "a second measure of competitiveness", according to Richardson, 1971a, p.236), while others found it of no economic meaning (see, for example, Reymert and Schultz, 1985).

In this paper we examine the CMS analysis within the context of index number theory and demonstrate that much of the criticism on this method can be clarified and partly overcome by reformulating the discrete–time version of the accounting decomposition of export (share) changes in a more convenient way. Specifically, it is shown that the residual term, interpreted as interaction effect, is due only to the formulas traditionally used and does not appear if a more flexible CMS decomposition method is adopted. As far as we know, no attempt has hitherto been made to find a solution to the CMS problem in the light of the developments of index number theory. We also demonstrate that other apparent pitfalls of the traditional applications of the method, such as, for example, the variability of the results depending on the order of decomposition of the single elements, are attributable to the particular decomposition formulas used rather than to a real drawback of the method itself.

In the next section we examine the traditional approaches starting from basic identities. In the third section we take into account the recent developments of index number theory which are useful for defining the CMS analysis in the discrete time in a way more consistent with its continuous–time version. In the fourth section a new formulation of CMS analysis is proposed by using Diewert's (1976) superlative index numbers. It is shown, in particular, that the residual "interaction" element is practically ruled out, being decomposed and distributed to the other CMS components. The fifth section concludes.

2. Basic identities and the traditional approaches of CMS analysis

Let us begin with the basic identity:

$$q^t \equiv \Sigma_i q_i^t = \Sigma_i s_i^t Q_i^t \tag{1}$$

where q^t = total exports of the focus country,

q_i^t = exports of the ith commodity of the focus country,

Q_i^t = world exports of the ith commodity,

$s_i^t \equiv q_i^t / Q_i^t$ = export share in the ith commodity of the focus country,

and superscript t denotes time.

The simplest formulation of CMS analysis can be obtained by differentiating (1) with respect to time:

$$\frac{dq^t}{dt} = \underbrace{\Sigma_i\, s_i^t \frac{dQ_i^t}{dt}}_{\text{Export growth with constant market shares}} + \underbrace{\Sigma_i\, Q_i^t \frac{ds_i^t}{dt}}_{\text{"Competitiveness" effect}} \tag{2}$$

In identity (2) export growth $(\frac{dq^t}{dt})$ is decomposed into two elements: a world growth effect $(\Sigma_i s_i^t \frac{dQ_i^t}{dt})$, which represents the overall export growth of the focus country in the presence of *constant market shares* in its exports of each commodity, and the so–called "competitiveness" effect $(\Sigma_i Q_i^t \frac{ds_i^t}{dt})$, which measures export changes due only to changes in market shares of that country.

An alternative formulation of CMS analysis starts with the following identity defining the focus country's aggregated export share:

$$(3) \qquad s^t = \Sigma_i \, s_i^t \, S_i^t$$

where s^t = aggregated export share of the focus country
S_i^t = share of the ith commodity in the world exports
$(S_i^t \equiv Q_i^t / \Sigma_i \, Q_i^t)$

Differentiating (3) with respect to time we obtain:

$$(4) \qquad \frac{ds^t}{dt} = \underbrace{\Sigma_i \, s_i^t \frac{dS_i^t}{dt}}_{\text{Structural effect}} + \underbrace{\Sigma_i \, S_i^t \frac{ds_i^t}{dt}}_{\text{"Competitiveness" effect}}$$

In identity (4) the growth in aggregated export share $(\frac{ds^t}{dt})$ is decomposed into two elements: a structural effect due to changes in commodity shares in the world trade $(\Sigma_i s_i^t \frac{dS_i^t}{dt})$, which reflects the export growth of the focus country in the hypothesis of its constant market shares, and the so–called "competitiveness" effect $(\Sigma_i S_i^t \frac{ds_i^t}{dt})$, which measures the changes of the focus country's aggregated export share due only to export share changes in each commodity.

Since identities (2) and (4) refer to continuous–time changes, they cannot be directly applied to discrete–time observations. In the literature on CMS analysis there is not a uniform way of translating the continuous–time into the discrete–time formulation. Tyszynski (1951) and Svennilson (1954) based their CMS decomposition formula on identity (3) and used the following decomposition:

$$(5) \qquad \Delta s = \underbrace{\Sigma_i \, s_i^0 \, \Delta S_i}_{\text{Structural effect}} + \underbrace{\Sigma_i \, S_i^1 \, \Delta s_i}_{\text{"Competitiveness" effect}}$$

Baldwin (1958), following Lichtenberg's suggestions to a previous draft of his paper, recognized that identity (5) amounts to using year 0 weights to measure the structural effects at constant market shares and year 1 weights to compute the "competitiveness" component. Therefore, he proposed the following alternative decomposition:

$$(5') \qquad \Delta s = \Sigma_i s_i^0 \, \Delta S_i + \Sigma_i S_i^0 \, \Delta s_i + \Sigma_i \Delta s_i \, \Delta S_i$$

where he noted that employing year 0 weights to compute both the competitive and structural effects leaves a residual term equal to $\Sigma_i \Delta s_i \Delta S_i$, which he called "interaction between the structural and competitive change" or simply "interaction effect" (see also Spiegelglas, 1959 for a similar procedure).

Richardson (1971a), focusing the attention on identity (1), proposed the following alternative decompositions:

$$(6) \qquad \Delta q = \Sigma_i s_i^0 \, \Delta Q_i + \Sigma_i Q_i^1 \, \Delta s_i$$

$$(7) \qquad \Delta q = \Sigma_i s_i^1 \, \Delta Q_i + \Sigma_i Q_i^0 \, \Delta s_i$$

$$(8) \qquad \Delta q = \Sigma_i [\alpha \, s_i^0 + (1 - \alpha) \, s_i^1] \, \Delta Q_i$$

$$+ \; \Sigma_i [(1 - \alpha) \, Q_i^0 + \alpha \, Q_i^1] \, \Delta s_i, \quad \text{for } 0 < \alpha < 1.$$

$$(9) \qquad \Delta q = \Sigma_i s_i^0 \, \Delta Q_i + \Sigma_i Q_i^0 \, \Delta s_i + \Sigma_i \Delta s_i \, \Delta Q_i$$

to which one could add

$$(10) \qquad \Delta q = \Sigma_i s_i^1 \, \Delta Q_i + \Sigma_i Q_i^1 \, \Delta s_i - \Sigma_i \Delta s_i \, \Delta Q_i$$

Identities (6) through (10) differ in the weights applied to each component. For example, (9) and (10) use a Laspeyres– and Paasche–type systems of weights respectively, while (6) and (7) use a Laspeyres–type for one component and a Paasche–type for another in order to assure consistency in the accounting for changes in total exports. Identity (8) is a combination of (6) and (7) or of (9) and (10) and uses symmetric weights. Richardson (1971a) does not acknowledge this aspect, which, as we shall demonstrate, makes (8) in some sense superior to the others. In fact, Richardson's (1971a) contention is opposite to ours: "no particular one of the identities (6) through

(9) has an a priori superiority to any other" (p.234). Therefore, he suggested to use them jointly because "CMS calculations which use only one of the available identities waste resources" (p.235). However, he recognizes that "CMS effects derived from identity (8) alone for some specified value of *a* may be useful if some representative numbers are desired for the period as a whole[3]. For example, if the researcher has no reason to believe that either the beginning– or end–of–period export structure was dominant throughout the period, CMS effects derived from identity (8) with α equal to 0.5 might be considered most representative of the period. This idea has particular application for studies in which the competitive effect has been regressed on relative price changes to obtain estimates of the elasticity of substitution. Since price changes occur *over* a period, the representative competitive effect *over* the period should be used, not the traditional competitive effect"(pp. 236–237).

The "residual" term $\Sigma_i \Delta s_i \Delta Q_i$ in (9) was interpreted by Richardson (1971a, p.236) as a "second measurement of 'competitiveness'", since it would indicate "whether the country was increasing its export shares in rapidly *growing* commodities and markets". He also noted that neither Baldwin (1958) nor Spiegelglas (1959) realized this potential interpretation. On the other hand, we can observe that this residual term is not present in the continuous–time decomposition. In fact the exact decomposition of total export change between period 0 and period 1 is expressed by integrating (2) over the interval between period 0 and period 1:

$$q^1 - q^0 = \int_0^1 \left[\Sigma_i s_i^t \frac{dQ_i^t}{dt}\right] dt + \int_0^1 \left[\Sigma_i Q_i^t \frac{ds_i^t}{dt}\right] dt \qquad (11)$$

Similarly, by integrating (4) we get the following exact decomposition of total export shares between period 0 and period 1:

$$s^1 - s^0 = \int_0^1 \left[\Sigma_i s_i^t \frac{dS_i^t}{dt}\right] dt + \int_0^1 \left[\Sigma_i S_i^t \frac{ds_i^t}{dt}\right] dt \qquad (12)$$

In Richardson's (1971a) words: "the problem is that over the time period under consideration, both a country's export structure and world exports *are continuously changing*. The typical researcher, however, has observations on only beginning– and end–of–period variables, while he optimally would like to know s_i^t and Q_i^t at every moment during the period" (p.234; italics added and mathematical symbols adjusted). We shall refer hereafter to this problem as the "index number problem of CMS analysis" or simply as "CMS problem", which we shall try to solve in the following sections. In particular, we shall demonstrate that the formulation of CMS analysis defined by (8), where $a = 0.5$, is the most accurate discrete–time approximation to (11) and permits us to decompose also the residual "interaction" effect.

An alternative formulation of CMS analysis would be in terms of relative rather than absolute changes. This is obtained by dividing identity (2) and (4) by $\Sigma_i s_i^t Q_i^t$ and $\Sigma_i s_i^t S_i^t$ respectively and integrating between periods 0 and 1, so that:

$$(13) \qquad \frac{q^1}{q^0} = \exp \int_0^1 \left[\Sigma_i \frac{s_i^t Q_i^t}{\Sigma_j s_j^t Q_j^t} \cdot \frac{dQ_i^t/dt}{Q_i^t} \right] dt \cdot \exp \int_0^1 \left[\Sigma_i \frac{s_i^t Q_i^t}{\Sigma_j s_j^t Q_j^t} \cdot \frac{ds_i^t/dt}{s_i^t} \right] dt$$

and

$$(14) \qquad \frac{s^1}{s^0} = \exp \int_0^1 \left[\Sigma_i \frac{s_i^t S_i^t}{\Sigma_j s_j^t S_j^t} \cdot \frac{dS_i^t/dt}{S_i^t} \right] dt \cdot \exp \int_0^1 \left[\Sigma_i \frac{s_i^t S_i^t}{\Sigma_j s_j^t S_j^t} \cdot \frac{ds_i^t/dt}{s_i^t} \right] dt$$

The discrete–time approximation of these decomposition formulas presents, however, similar problems as those with (11) and (12).

Another serious criticism raised by Richardson (1971a,1971b) on CMS analysis regards the decomposition of "structural" effects proposed in later versions of the method. Taking into account the market distribution of the focus country's exports of each commodity, identity (1) becomes:

(15) $q^t = \Sigma_i \Sigma_j s^t_{ij} Q^t_{ij}$

where $s^t_{ij} \equiv q^t_{ij}/Q^t_{ij}$ and j denotes the jth market.

Differentiating and expanding (15), the basic CMS decomposition can be re–expressed as:

(16) $$\frac{dq^t}{dt} = \underbrace{s^t \frac{dQ^t}{dt}}_{\text{World growth effect}} + \underbrace{\left[\Sigma_i s^t_{i.} \frac{dQ^t_{i.}}{dt} - s^t \frac{dQ^t}{dt}\right]}_{\text{Commodity effect}}$$

$$+ \underbrace{\left[\Sigma_i \Sigma_j s^t_{ij} \frac{dQ^t_{ij}}{dt} - \Sigma_i s^t_{i.} \frac{dQ^t_{i.}}{dt}\right]}_{\text{Market effect}} + \underbrace{\Sigma_i \Sigma_j \frac{ds^t_{ij}}{dt} Q^t_{ij}}_{\text{Competitive effect}}$$

or, alternatively,

(17) $$\frac{dq^t}{dt} = \underbrace{s^t \frac{dQ^t}{dt}}_{\text{World growth effect}} + \underbrace{\left[\Sigma_j s^t_{.j} \frac{dQ^t_{.j}}{dt} - s^t \frac{dQ^t}{dt}\right]}_{\text{Market effect}}$$

$$+ \underbrace{\left[\Sigma_i \Sigma_j s^t_{ij} \frac{dQ^t_{ij}}{dt} - \Sigma_j s^t_{.j} \frac{dQ^t_{.j}}{dt}\right]}_{\text{Commodity effect}} + \underbrace{\Sigma_i \Sigma_j \frac{ds^t_{ij}}{dt} Q^t_{ij}}_{\text{Competitive effect}}$$

where $Q^t \equiv \Sigma_i \Sigma_j Q^t_{ij}$, $s^t \equiv q^t/Q^t$, $s^t_{i.} \equiv \Sigma_j q^t_{ij}/\Sigma_j Q^t_{ij}$, $s^t_{.j} \equiv \Sigma_i q^t_{ij}/\Sigma_i Q^t_{ij}$, $Q^t_{i.} \equiv \Sigma_j Q^t_{ij}$, $Q^t_{.j} \equiv \Sigma_i Q^t_{ij}$. In identities (16) and (17) the export growth effects with constant market shares are subdivided into three elements: (i) the world growth effect, which is represented by the country's export change that will occur if the country's export growth in each commodity and market is equal to that of the world export, (ii) the commodity effect, and (iii) the market effect. These last two effects arise from the difference between the structural concentration of the focus country's exports on commodities and importing regions, respectively, and those of world trade average.

Richardson (1971b) made clear that, although the sum of the commodity and market effects in (16) would be no different from that calculated in (17), the change in the sequence of calculation would modify the values of the individual commodity and market effects. We can, in fact, have the following different measures:

$$\left[\Sigma_i s^t_{i.} \frac{dQ^t_{i.}}{dt} - s^t \frac{dQ^t}{dt}\right] \neq \left[\Sigma_i \Sigma_j s^t_{ij} \frac{dQ^t_{ij}}{dt} - \Sigma_j s^t_{.j} \frac{dQ^t_{.j}}{dt}\right]$$

for commodity effect, and

$$\left[\Sigma_i \Sigma_j s^t_{ij} \frac{dQ^t_{ij}}{dt} - \Sigma_i s^t_{i.} \frac{dQ^t_{i.}}{dt}\right] \neq \left[\Sigma_j s^t_{.j} \frac{dQ^t_{.j}}{dt} - s^t \frac{dQ^t}{dt}\right]$$

for market effect. This discrepancy corresponds to a difference of concepts implied by the formulas for commodity and market effects in (16) and (17). This will become more evident in the following discussion.

The same considerations apply to the well–known version of CMS analysis usually attributed to Leamer and Stern (1970), although Narvekar (1960–61) extensively used it ten years before (see also Stern, 1967, pp. 33–42, 161–64). In fact this version is a discrete–time formulation of (16) (which is of the same type of decomposition given by (6)):

$$\text{(18)} \quad \Delta q = \underbrace{s^0 \Delta Q}_{\text{World growth effect}} + \underbrace{(\Sigma_i s^0_{i.} \Delta Q_{i.} - s^0 \Delta Q)}_{\text{Commodity effect}}$$

$$+ \underbrace{(\Sigma_i \Sigma_j s^0_{ij} \Delta Q_{ij} - \Sigma_i s^0_{i.} \Delta Q_i)}_{\text{Market effect}} + \underbrace{(\Sigma_i \Sigma_j Q^1_{ij} \Delta s_{ij})}_{\text{Competitive effect}}$$

If we substitute $s^0 \Delta Q$ with $(s^0 Q^0) \cdot (\Delta Q / Q^0)$, $s^0_{i.} \Delta Q_{i.}$ with $(s^0_i Q^0_{i.}) \cdot (\Delta Q_{i.} / Q^0_{i.})$, $s^0_{ij} \Delta Q_{ij}$ with $(s^0_{ij} Q^0_{ij}) \cdot (\Delta Q_{ij} / Q^0_{ij})$, and $Q^1{}_j \Delta s_{ij}$ with $\left[\Delta(Q_{ij} s_{ij}) - (\Delta Q_{ij} / Q^0_{ij}) \cdot Q^0_{ij} s^0_{ij}\right]$, then identity (18) is expressed exactly in the form of the Leamer and Stern's (1970) version of CMS analysis.

More recently, Fagerberg and Sollie (1987) decomposed export share changes by implicitly using the following basic identity:

$$(19) \qquad s^t = \Sigma_i \Sigma_j \quad \underset{\substack{\text{Country's export}\\\text{share of the ith}\\\text{commodity in the}\\\text{jth market}}}{s^t_{ij}} \cdot \underset{\substack{\text{Share of the ith}\\\text{commodity in total}\\\text{imports of the jth}\\\text{market}}}{\frac{Q^t_{ij}}{\Sigma_i Q^t_{ij}}} \cdot \underset{\substack{\text{Share of the jth}\\\text{market in total}\\\text{world imports}}}{\frac{\Sigma_i Q^t_{ij}}{\Sigma_i \Sigma_j Q^t_{ij}}}$$

An alternative basic identity could be the following:

$$(20) \qquad s^t = \Sigma_i \Sigma_j \quad \underset{\substack{\text{Country's export}\\\text{share of the ith}\\\text{commodity in the}\\\text{jth market}}}{s^t_{ij}} \cdot \underset{\substack{\text{Share of the jth}\\\text{market in the}\\\text{world imports of}\\\text{the jth commodity}}}{\frac{Q^t_{ij}}{\Sigma_j Q^t_{ij}}} \cdot \underset{\substack{\text{Share of the ith}\\\text{commodity in total}\\\text{world imports}}}{\frac{\Sigma_j Q^t_{ij}}{\Sigma_i \Sigma_j Q^t_{ij}}}$$

The identity actually used by Fagerberg and Sollie (1987) is:

(21) $$s^t = \Sigma_j M_j^t c_j^t$$

where $M_j \equiv \Sigma_i s_{ij}^t \dfrac{Q_{ij}^t}{\Sigma_i Q_{ij}^t}$ (defined as "market share of the focus country in the country j's imports"), and $c_j^t \equiv \dfrac{\Sigma_i Q_{ij}^t}{\Sigma_i \Sigma_j Q_{ij}^t}$ (defined as "country j's share of world imports"). Starting from (21), they applied the following decompositions:

(22) $$\Delta s = \Sigma_j \Delta M_j c_j^0 + \Sigma_j M_j^0 \Delta c_j + \Sigma_j \Delta M_j \Delta c_j$$

and

(23) $$\Delta M_j = \Sigma_i s_{ij}^0 \Delta \left[\frac{Q_{ij}}{\Sigma_i Q_{ij}}\right] + \Sigma_i \Delta s_{ij} \frac{Q_{ij}^0}{\Sigma_i Q_{ij}^0} + \Sigma_i \Delta s_{ij} \cdot \Delta \left[\frac{Q_{ij}}{\Sigma_i Q_{ij}}\right]$$

Substituting (23) into the first additive term of the right–hand side of (22) leads to:

(24) $$\Delta s = \underbrace{\Sigma_j M_j^0 \Delta c_j}_{\text{Market–distribution effect}} + \underbrace{\Sigma_j \Sigma_i s_{ij}^0 \Delta \left[\frac{Q_{ij}}{\Sigma_i Q_{ij}}\right] c_j^0}_{\text{Commodity–composition effect}}$$

$$+ \underbrace{\Sigma_j \Sigma_i \Delta s_{ij} \frac{Q_{ij}^0}{\Sigma_i Q_{ij}^0} c_j^0}_{\text{Market–share effect ("Competitiveness" effect)}} + \underbrace{\Sigma_j \Sigma_i \Delta s_{ij} \Delta \left[\frac{Q_{ij}}{\Sigma_i Q_{ij}}\right] c_j^0}_{\text{Commodity–adaptation effect}}$$

$$+ \ \Sigma_j \ \ \Delta M_j \ \Delta c_j$$

Market–
adaptation
effect

Eq. (24) has been applied by OECD (1987, pp. 274–75), which computed the five effects for exports of the major industrial countries. The following comments can be made on this approach. Firstly, we note that the market–distribution and commodity–composition effects in the Fagerberg and Sollie (1987) version of CMS analysis given by (24) have different meaning and values with respect to those of the Narvekar (1960–61), Leamer and Stern (1970), Richardson (1971a, 1971b) version of CMS analysis given by (18). Fagerberg and Sollie (1987) did not make any reference to this difference in concepts between their own and the above–mentioned CMS version, except that they used Laspeyres indexes throughout the calculations.

In (24) the market and commodity effects are respectively seen as the effects on the country's export performance of the dynamic structural changes of its original export markets and of world trade in commodities and, therefore, can be considered as an extension of the "structural" effect concept in Tyszynski (1951), Baldwin (1958) and Spiegelglas (1959) analyses. In (18) they are explicitly seen as effects that arise from the different concentration of the country's exports in markets and commodities with growth rates more (less) favorable than world average. It can be easily shown, by means of algebraic manipulations, that eq. (24), based on identity (19), is a discrete–time export–share version of the CMS decomposition given by (17), while eq. (18) can be brought back to identity (20) and is a discrete–time version of decomposition given by (16). This sheds light on the difference in meaning between these two alternative CMS procedures, which has often been misunderstood in the literature.

Secondly, identity (24) is a Laspeyres–type decomposition formula which contains two "interaction effects", the commodity–adaptation and the market–adaptation effects, in addition to the traditional three components (commodity–composition, market–distribution and "competitiveness" effects)[4]. The commodity–adaptation effect is interpreted as the degree of success of the focus country in adapting the commodity composition of its exports to the overall change in the commodity composition of world imports, while the market–adaptation effect is interpreted as the degree of success of the focus country in adapting the market composition of its exports to the evolution of geographical distribution of world imports. Two interaction effects arise here because the decomposition of export shares involves more than two types of components. On the other hand, these are extraneous to the corresponding continuous–time formulation of CMS analysis.

Moreover, we observe that different formulas and values for each component can be found if a Paasche–type decomposition is applied. It is therefore evident that Fagerberg and Sollie's (1987) formulation of CMS analysis presents an index number problem which is similar to that of the other versions already discussed.

3. The CMS problem in the light of recent developments of index number theory

The CMS problem can be reinterpreted and clarified within the context of recent advances in index number theory. These will ultimately permit us to find a more satisfactory reformulation of CMS analysis without contradicting, however, the basic ideas of the early contributions in this field. It is in fact generally understood that the contemporary research in index number theory "offers a considerable commentary on formula specification which complements rather than contradicts the original work of the empiricists of the early part of this century" (Hansen and Lucas, 1984, p.25)[5].

The original continuous–time decomposition formulas (11) and (12) for CMS analysis are given by the sum of line integrals that were called the "Schumpeterian version" of Divisia indexes by Samuelson and Swamy (1974, p.578). The Divisia indexes are, in general, ***path–dependent***, that is they depend on the path over which the integration is taken. This is to say that, for the same initial and final values of the elementary components, the aggregating index depends, in general, on the history of these components during the time interval. Samuelson and Swamy (1974, pp.579–80) and Diewert (1981, pp.195–96) summarize many studies of the necessary and sufficient conditions under which the Divisia index is path–independent. These are shown to be equivalent to the existence and homotheticity conditions of the underlying aggregator function (see also Hulten, 1973).

When identity (11) or (12) is to be applied to data referring to discrete–time observations some way has to be found for approximating the Divisia line integrals. In the recent literature a number of approximating formulas have been developed and their properties have been studied with reference to the underlying aggregator functions. Basic contributions in the so–called "functional approach" to index number theory[6], due to Samuelson and Swamy (1974), Diewert (1976,1978), and Lau (1979) have clarified that certain functional forms for the aggregator function can be associated with certain index number formulas. In other words, these can be defined as *exact* for particular aggregator functions since they are equal to the ratio of the values of these functions at an initial and final point.

More specifically, the Laspeyres– or Paasche–type aggregating indexes are exact for aggregator functions (the Leontief–type fixed coefficient functions) which are first–order numerical approximations to the true aggregator functions. Similarly, the so–called *geometric indexes* are exact for Cobb–Douglas–type aggregator functions and belong to the family of indexes which are exact for first–order approximating functions.

Another class of index numbers is very important for their property of being exact for aggregator functions which can provide a second–order differential approximation to arbitrary twice–differentiable aggregator functions and have at the same time convenient mathematical properties. These indexes were called *superlative* by Diewert (1976) (who quoted Fisher, 1922, p.247 for the use of an undefined notion of this term). Most of superlative index numbers can be derived as special cases from the *Quadratic mean of order r index* (which is itself a superlative index number). The Törnqvist and the Fisher ideal belong to this class of index numbers: the former being exact for the Translog aggregator function, the latter being exact for the Quadratic of order 2 aggregator function. The Törnqvist index is usually used to approximate the Divisia line integrals with discrete–time observations (see, for example, Theil, 1965,1967, Christensen and Jorgenson, 1970, Jorgenson and Griliches, 1972, Star, 1974, Star and Hall, 1976, Barnett, 1980, Barnett, Offenbacher and Spindt, 1984, Vartia and Vartia, 1984).

These developments permit us to reinterpret the CMS decomposition formulas in terms of index numbers which are exact for some kind of aggregator functions. In fact, if we consider the last identity in (1) and apply the Taylor's series expansion of q^1 around the point (s^0,Q^0) (where $s^0 \equiv [s_1^0, s_2^0, \ldots,s_n^0]^T$ and $Q^0 \equiv [Q_1^0, Q_2^0, \ldots,Q_n^0]^T$), we get just the decomposition formula (9). Here the term $\Sigma_i \Delta s_i \Delta Q_i$ can be interpreted as the second–order remainder term of the linear approximation given by the first–order Taylor's series expansion. Similarly, if we apply the Taylor's series expansion of q^0 around the point (s^1,Q^1), we get just the decomposition formula (10), where the term $-\Sigma_i \Delta s_i Q_i$ can be interpreted as the second–order remainder term of a linear approximation. In this view the so–called "interaction effects", in Baldwin's (1958) and Spiegelglas' (1959) terminology, is produced by the inability of the linear approximating formula to completely disentangle the component effects by tracing a non–linear function.

By contrast, taking account of the fact that $q^t = \Sigma_i s_i^t Q_i^t$ is a quadratic function in the s_i^t's and Q_i^t's, we can apply the *Quadratic lemma* (see Diewert, 1976, p.118) which states that, for any quadratic function $y = f(z_1, z_2,...,z_m)$ such that

$$(25) \qquad f(z_1, z_2,...,z_m) \equiv a_0 + \Sigma_i a_i z_i + \Sigma_i \Sigma_j a_{ij} z_i z_j$$

where a_i, a_{ij} are constants and $a_{ij} = a_{ji}$ for all i,j, then

$$(26) \qquad y^1 - y^0 = \Sigma_i \frac{1}{2}\left[\frac{\partial f(z^0)}{\partial z_i^0} + \frac{\partial f(z^1)}{\partial z_i^1}\right](z_i^1 - z_i^0)$$

This should be contrasted with the Taylor's expansion series which could be applied to obtain (9) and (10), where the first and second—order partial derivatives *evaluated at an initial point* are used. In the expansion (26) only first–order partial derivatives are required, but they must be *evaluated at both initial and final points.* Denny and Fuss (1983) have shown that (26) can be considered as a *growth accounting equation*, which can also be obtained by subtracting the two linear approximations given by the Taylor's series expansions around the initial and final points and then dividing through by 2.

Since $q^t = \Sigma_i s_i^t Q_i^t$ is a special case of (25), the corresponding growth accounting equation for the interval between period 0 and period 1 can be written as:

$$(27) \qquad \Delta q = \Sigma_i \frac{1}{2}\left[s_i^0 + s_i^1\right] \Delta Q_i + \Sigma_i \frac{1}{2}\left[Q_i^0 + Q_i^1\right] \Delta s_i$$

which corresponds to (8) when we set $\alpha = 0.5$ in this last identity.

We note that (27) is a Törnqvist–type index as it takes the average of weights in the initial and final points (eq. (26) would lead to the logarithm of the Törnqvist index if all the variables were expressed in logarithmic terms). The intrinsic nature of (27) makes it superior to those CMS decomposition formulas which can be interpreted as first–order numerical approximations.

We may ask, however, under what conditions the line integrals in (11) correspond exactly to the respective additive terms in (27). If, by chance, the s_i^t's and the Q_i^t's follow a path which is consistent with aggregator functions $\phi(s^t)$ and $\varphi(Q^t)$ and, moreover, the duality relationships $\frac{\partial \phi(s^t)}{\partial s_i^t} = Q_i^t$ and $\frac{\partial \varphi(Q^t)}{\partial Q_i^t} = s_i^t$ are always true, then by the Quadratic lemma a sufficient condition is that $\phi(s^t)$ and $\varphi(Q^t)$ are quadratic functions for the above–mentioned equality to hold. Furthermore, the invariance of the line integrals in (11) under change of path between endpoints is guaranteed if and only if $\phi(s^t)$ and $\varphi(Q^t)$ are homothetic functions.

In the general case, the focus country's export shares and world exports are highly unlikely to follow a path satisfying the above conditions. Using eq. (27) can thus bring about, at best, approximation errors of some magnitude. However, Samuelson and Swamy (1974, p. 579, n.11) and Star and Hall (1976) have shown that, if there are data for intervals "close together" between endpoints, the approximation error can be substantially reduced by applying the Tõrnqvist index interval by interval and then constructing the overall index by chaining (see also Diewert, 1978, 1981, Trivedi, 1981, and Hill, 1988 for strong justification of the *chain principle*).

When the CMS analysis is to be expressed in terms of relative changes as specified by eq. (13) or (14), we can observe that the line integrals of the decomposition equation are Divisia indexes. They can be approximated by direct and implicit Tõrnqvist indexes by using discrete–time observations. In the case we use eq. (13) we obtain:

$$\text{(28)} \qquad \ln q^1 - \ln q^0 = \ln \bar{s}_{0,1} + \ln Q_{0,1}$$

where $\ln Q_{0,1} \equiv \Sigma_i \frac{1}{2}\left[\frac{s_i^0 Q_i^0}{\Sigma_j s_j^0 Q_j^0} + \frac{s_i^1 Q_i^1}{\Sigma_j s_j^1 Q_j^1}\right] (\ln Q_i^1 - \ln Q_i^0)$ and

$$\ln \bar{s}_{0,1} \equiv (\ln q^1 - \ln q^0) - \Sigma_i \frac{1}{2}\left[\frac{s_i^0 Q_i^0}{\Sigma_j s_j^0 Q_j^0} + \frac{s_i^1 Q_i^1}{\Sigma_j s_j^1 Q_j^1}\right] (\ln Q_i^1 - \ln Q_i^0).$$

In eq. (28) $Q_{0,1}$ is the direct Tõrnqvist index of the world export growth effects, while $\bar{s}_{0,1}$ is the implicit Tõrnqvist index of the "competitiveness" effect. Since the Tõrnqvist index does not satisfy the factor reversal test exactly, one of the aggregated components has to be computed *implicitly* from the relative change of the focus country's exports and the direct Tõrnqvist index of the other components.

4. The reformulation of CMS analysis

In the light of index number theory we find that the traditional approaches of CMS analysis should not be used for the following related reasons:

1) The underlying Laspeyres– or Paasche–type aggregating formulas give a residual term which can be interpreted as a linear approximation error of a second–order magnitude.

2) The Laspeyres– and Paasche–type components are in general significantly biased.

The discrete–time approximations based on the so–called superlative index numbers must be preferred since they may lead to approximation errors of third– or higher–order magnitude. One of these index numbers is the Tõrnqvist index, which is exact for an aggregating Translog function.

In the empirical application of the Tõrnqvist–type indexes we should take advantage of the following suggestions. It has been recognized that the shorter the time period and the smaller the rates of change the closer the discrete Tõrnqvist approximation to the Divisia index. It is therefore strongly recommended to apply the chain principle by subdividing the whole time period into the shortest intervals for which the data are available and to use chained indexes to reconstruct the accounting decomposition.

Furthermore, in the construction of CMS index numbers of aggregate components of relative changes we have to decide which are to be estimated by the *direct* Tõrnqvist index and which is the one that is to be derived implicitly. Allen and Diewert (1981) have argued that the elementary components that show more highly proportional variations have to be aggregated through the *direct* superlative index numbers, while the *implicit* superlative indexes are to be preferred for the aggregator of the other components. In this way the direct superlative index numbers are more likely to satisfy the basic theoretical approximation properties and to be closer to the true (unknown) aggregating indexes.

In the case where CMS analysis has to be expressed in terms of absolute changes of the country's exports we can apply the discrete–time decomposition (27) to (16) or (17). The discrete–time decomposition of (16) is derived as follows:

(29) $$\Delta q = \frac{1}{2}\left[s^0 + s^1\right]\Delta Q$$

World growth effect

$$+ \Sigma_i \frac{1}{2}\left[s^0_{i.} + s^1_{i.}\right] \cdot \Delta Q_i - \frac{1}{2}\left[s^0 + s^1\right]\Delta Q$$

Commodity–composition effect

$$+ \Sigma_i\Sigma_j \frac{1}{2}\left[s^0_{ij} + s^1_{ij}\right]\cdot\Delta Q_{ij} - \Sigma_i \frac{1}{2}\left[s^0_{i.} + s^1_{i.}\right]\Delta Q_{i.}$$

Market–distribution effect

$$+ \Sigma_i\Sigma_j \frac{1}{2}\left[Q^0_{ij} + Q^1_{ij}\right]\Delta s_{ij}$$

"Competitiveness" effect

The decomposition (29) is our proposed discrete–time version of (16). It is also a reformulation of the traditional decomposition (18), which is equivalent to the Narvekar (1960–61), Leamer and Stern (1970), Richardson (1971a, 1971b) version of CMS analysis. We note that in the analogous discrete–time version of (17) the components that can be identified as market–distribution and commodity–composition effects have different values and meaning with respect to those of (29).

The more recent CMS decomposition (24) used by Fagerberg and Sollie (1987) and OECD (1987) can be similarly reformulated starting from identity (19) or (21) as follows:

(30) $$\Delta s = \Sigma_j \frac{1}{2}\left[M^0_j + M^1_j\right]\Delta c_j$$

$$+ \Sigma_j \frac{1}{2}\left[c^0_j + c^1_j\right]\Delta M_j$$

and

(31)
$$\Delta M_j = \Sigma_i \frac{1}{2}\left[\frac{Q^0_{ij}}{\Sigma_i Q^0_{ij}} + \frac{Q^1_{ij}}{\Sigma_i Q^1_{ij}}\right]\Delta s_{ij}$$

$$+ \Sigma_i \frac{1}{2}\left[s^0_{ij} + s^1_{ij}\right]\Delta\left[\frac{Q_{ij}}{\Sigma_i Q_{ij}}\right]$$

Substituting (31) into (30), we get:

(32)
$$\Delta s = \Sigma_j \frac{1}{2}\left[M^0_j + M^1_j\right]\Delta c_j$$

Market–distribution effect

$$+ \Sigma_i\Sigma_j \frac{1}{2}\left[s^0_{ij} + s^1_{ij}\right]\Delta\left[\frac{Q_{ij}}{\Sigma_i Q_{ij}}\right]\frac{1}{2}\left[c^0_j + c^1_j\right]$$

Commodity–composition effect

$$+ \Sigma_i\Sigma_j \Delta s_{ij} \frac{1}{2}\left[\frac{Q^0_{ij}}{\Sigma_i Q^0_{ij}} + \frac{Q^1_{ij}}{\Sigma_i Q^1_{ij}}\right]\frac{1}{2}\left[c^0_j + c^1_j\right]$$

"Competitiveness" effect

We observe that the market–adaptation and commodity–adaptation effects in the original Fagerberg and Sollie (1987) formulation given by (24) are not present in eq. (32), where they are decomposed and distributed to the other components by a more flexible procedure. Furthermore, the concepts and values of market–distribution and commodity–composition effects in eq. (32) are different from those given by eq. (29). In (32) the market–distribution and commodity–composition effects are seen as arising, respectively, from the growth in the country's export markets and in the world trade of commodities in each market, while in (29) they are seen as arising from the concentration of the country's exports in markets and commodities with growth rates more (less) favorable than world average. At a closer examination, it can be shown that eq. (32) is the discrete–time (export–share) version of the CMS decomposition (17), being based on identity (19), while eq. (29) is the discrete–time version of the

CMS decomposition (16) and can be derived starting from an identity similar to (20). Specifically, in eqs. (17) and (32) the market—distribution effect refers to the change in the export market distribution in the world trade of all commodities, while in eqs. (16) and (29) it refers to the change in the export market distribution in the world trade of each specific commodity. Moreover, in eqs. (17) and (32) the commodity–composition effect refers to the change in the relative importance of commodities in the imports of each specific market, while in eqs. (16) and (29) it refers to the change in the relative importance of commodities in the total world imports (or exports).

Finally, if the CMS analysis has to be specified in terms of relative changes, we can apply a discrete–time approximation formula similar to (28) starting from identity (19) or (20), thus obtaining:

$$(33) \qquad \Delta \ln q = \ln \bar{s}_{0,1} + \ln C_{0,1} + \ln M_{0,1} + \Delta \ln Q$$

where

$$(34) \qquad \ln \bar{s}_{0,1} \equiv \Delta \ln q - \ln C_{0,1} - \ln M_{0,1} - \Delta \ln Q$$

which is the logarithm of the implicit Tõrnqvist index of the "competitiveness" effect and, if identity (19) is used,

$$(35) \quad \ln C_{0,1} \equiv \Sigma_i \Sigma_j \frac{1}{2} \left[\frac{s^0_{ij} \cdot Q^0_{ij}}{\Sigma_i \Sigma_j s^0_{ij} Q^0_{ij}} + \frac{s^1_{ij} \cdot Q^1_{ij}}{\Sigma_i \Sigma_j s^1_{ij} Q^1_{ij}} \right] \Delta \ln \frac{Q_{ij}}{\Sigma_i Q_{ij}}$$

which is the logarithm of the direct Tõrnqvist index of commodity—composition effects, while

$$(36) \quad \ln M_{0,1} \equiv \Sigma_i \Sigma_j \frac{1}{2} \left[\frac{s^0_{ij} \cdot Q^0_{ij}}{\Sigma_i \Sigma_j s^0_{ij} Q^0_{ij}} + \frac{s^1_{ij} \cdot Q^1_{ij}}{\Sigma_i \Sigma_j s^1_{ij} Q^1_{ij}} \right] \Delta \ln \frac{\Sigma_i Q_{ij}}{\Sigma_i \Sigma_j Q_{ij}}$$

which is the logarithm of the direct Tõrnqvist index of market—distribution effect. Here the "competitiveness" effect is estimated through the implicit Tõrnqvist index number. However, if the available data show a higher proportionality of the changes in the focus country's export shares than in the other elementary components, the direct Tõrnqvist index number must be used, while in this case the implicit Tõrnqvist index is more appropriate for commodity effects or market effects. We note, also, that the two last effects have a meaning similar to that in the decomposition given by eq. (32).

If identity (20) is used instead of (19), then the terms

$$\left[\Delta \ln \frac{Q_{ij}}{\Sigma_i Q_{ij}}\right] \text{ and } \left[\Delta \ln \frac{\Sigma_i Q_{ij}}{\Sigma_i \Sigma_j Q_{ij}}\right]$$

in (35) and (36) must be substituted by

$$\left[\Delta \ln \frac{Q_{ij}}{\Sigma_j Q_{ij}}\right] \text{ and } \left[\Delta \ln \frac{\Sigma_j Q_{ij}}{\Sigma_i \Sigma_j Q_{ij}}\right]$$

respectively. With these substitutions $\ln C_{0,1}$ is the logarithm of the direct Tõrnqvist index of market–distribution effect, while $\ln M_{0,1}$ is the logarithm of the direct Tõrnqvist index of commodity–composition effect. Here again, we recall that the value and meaning of these market–distribution and commodity–composition effects are different from those derived on the basis of identity (19).

5. *Conclusion*

In this paper we have revisited the major accounting decomposition problems in Constant–Market–Shares Analysis. These were recognized as index number problems in the early empirical applications. By interpreting the alternative CMS decomposition formulas in the light of recent developments in index number theory we were able to identify the unresolved aspects of the decomposition procedure and to propose satisfactory solutions at least from a theoretical viewpoint. These solutions would significantly improve the quality of the empirical analysis and increase the readability of the results obtained.

A decomposition analysis should not use, in general, the fixed—share weighted Laspeyres– or Paasche–type indexes when the weights are not constant during the time interval under examination. More "flexible" index numbers should be used instead, paying attention to the particular dynamics of the elementary components and to the underlying law of aggregation established by some particular aggregator function. When the functional forms of these aggregator functions are not known, the most suitable indexes are Diewert's superlative index numbers which are exact for aggregator quadratic functions. One of these indexes, the Tõrnqvist index number, is characterized by many desirable properties. It is exact for the Translog aggregator function and can be used as a good discrete–time approximation to the Divisia index. This last property turns out to be very convenient since CMS analysis can be directly translated into a decomposition procedure based on Divisia indexes.

Our proposed reformulation of CMS analysis is very simple in nature and does not show the pitfalls of the traditional approaches, such as the residual that arises when a linear formula is applied to approximate a non–linear function. More specifically, it is based on the Quadratic lemma, which permits us to decompose absolute or relative changes of a function by using quadratic approximating functions either directly or indirectly through the corresponding superlative index numbers. The decomposition bias is thus significantly reduced if compared with that arising from the traditional approaches. Moreover, if the chain principle is applied by subdividing the time interval into the shortest periods of time for which the data are available, the bias is further reduced.

It must be conceded, however, that it is unlikely that variations of the elementary components over time are *precisely* consistent with an aggregator quadratic function and, consequently, the bias cannot be completely eliminated. As Samuelson and Swamy (1974, p.592) put it, "we must accept the sad facts of life, and be grateful for the more complicated procedures economic theory devises".

Notes:

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[1] See Creamer (1943) for one of the first empirical applications and Houston (1967) and Stevens and Moore (1980) for critical reviews of the literature in these fields.

[2] For a list of early studies in international economics which used CMS analysis see Richardson (1971a, p.227, n. 1) and, for references to recent contributions, Fagerberg and Sollie (1987, p.1572, n.2).

[3] Identity (8) is also mentioned by Magee (1968, p.37) as a possible solution to the index number problem when $a = 0.5$, since then the weights on ΔQ_i and Δs_i will be "consistent", or similarly dimensioned.

[4] It would also be possible to develop a Paasche—type decomposition formula starting from (21).

[5] See Allen (1975), Diewert (1981, 1986), Jazairi (1983), and Hill (1988) for surveys on the developments of index number theory.

[6] Frisch (1936) distinguishes three approaches to index numbers theory: (i) descriptive or statistical approach, (ii) the test approach, and (iii) the functional approach. The third one was also called the "economic theory of index numbers" by Staehle (1934–35), Samuelson (1947, pp.146–56), Samuelson and Swamy (1974, p.573), and Diewert (1981, 1986).

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Summary. This paper examines the constant–market–shares (CMS) analysis of a country's export growth within the context of index number theory and is aimed at finding a satisfactory solution to the problems encountered by the traditional CMS decomposition procedures. The last ones perform the task of disentangling the variations of each accounting factor only partially, because they are based on linear approximations to non–linear functions. The residual "interaction" term left out in the current CMS analyses does not appear if a more flexible CMS decomposition based on the so–called *superlative* index numbers is used. Moreover, the discussion of the basic identities has clarified the difference in meaning and levels of the accounting components between the alternative versions of CMS analysis.